Anisotropic microwave conductivity of cuprate superconductors in the presence of CuO chain-induced impurities

Zhi Wang and Shiping Fen[g*](#page-5-0)

Department of Physics, Beijing Normal University, Beijing 100875, China (Received 6 August 2009; published 6 November 2009)

The anisotropy in the microwave conductivity of the ortho-II $YBa₂Cu₃O_{6.50}$ is studied within the kineticenergy driven superconducting mechanism. The ortho-II YBa₂Cu₃O_{6.50} is characterized by a periodic alternative of filled and empty \hat{b} -axis CuO chains. By considering the CuO chain-induced extended anisotropy impurity scattering, the main features of the anisotropy in the microwave conductivity of the ortho-II $YBa₂Cu₃O_{6.50}$ are reproduced based on the nodal approximation of the quasiparticle excitations and scattering processes, including the intensity and line shape of the energy and temperature dependences of the *aˆ*-axis and \hat{b} -axis microwave conductivities. Our results also confirm that the \hat{b} -axis CuO chain-induced impurity is the main source of the anisotropy.

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I. INTRODUCTION

Impurity plays a crucial role in determining the behavior of many measurable properties of cuprate superconductors¹ such as the quasiparticle transport in the superconducting (SC) state. This follows from a fact that the physical properties of cuprate superconductors in the SC state are extreme sensitivity to the impurity effect than the conventional superconductors due to the finite angular momentum chargecarrier Cooper pairing with the d -wave symmetry.¹ The single common feature is the presence of the $CuO₂$ plane² and it seems evident that the unusual behaviors of cuprate superconductors are dominated by this $CuO₂$ plane.³ Experimentally, over 20 years measurements of electrodynamic properties at microwave energies have provided rather detailed information on the quasiparticle transport of cuprate superconductors, $4-9$ where the nodal quasiparticle spectrum contains most of the features expected for weak-limit impurity scattering. The early microwave-conductivity measurements $4\overline{4}$ showed the microwave-conductivity spectrum is $CuO₂$ -plane isotropic. However, the recent development of a sufficient number of fixed-energy cavity perturbation system to map out a coarse microwave-conductivity spectrum^{8,[9](#page-5-5)} allowed to resolve additional features in the microwave-conductivity spectrum. Among these achievements is the observation of an anisotropy in the microwaveconductivity spectrum of the highly ordered ortho-II $YBa₂Cu₃O_{6.50}$, where the well-pronounced difference in intensity and line shape in the \hat{a} -axis and \hat{b} -axis directions is remarkable.

Although the anisotropy in the microwave-conductivity spectrum of the ortho-II YBa₂Cu₃O_{6.50} is well-established experimentally, 8.9 its full understanding is still a challenging issue. Theoretically, based on a phenomenological Bardeen-Cooper-Schrieffer (BCS) formalism with the *d*-wave SC gap function, it has been argued in the mean-field level that the interband transitions produce a strongly anisotropic feature.¹⁰ However, it has been shown that the $CuO₂$ bilayer may lead to two nearly identical contributions to both *aˆ*-axis and \hat{b} -axis microwave conductivities⁸ since the splitting between

bonding and antibonding combinations of the planar wave functions is expected to be weak.¹¹ In our earlier work¹² based on the kinetic-energy driven SC mechanism, $13,14$ $13,14$ the effect of the extended impurity-scattering potential on the quasiparticle transport in the nonortho-II phase of cuprate superconductors has been discussed within the nodal approximation of the quasiparticle excitations and scattering processes, and the obtained energy and temperature dependences of the microwave conductivity are consistent with the experimental data in the nonortho-II phase of cuprate superconductors. $4-7$ In the *d*-wave SC state of cuprate superconductors, the characteristic feature is the existence of four nodal points $\left[\pm \frac{\pi}{2}, \pm \frac{\pi}{2}\right]$ (in units of inverse lattice constant) in the Brillouin zone, $2,15$ $2,15$ where the SC gap function vanishes, therefore the $CuO₂$ -plane currents are mainly carried by nodal quasiparticles and the quasiparticle transport properties of cuprate superconductors in the SC state are largely governed by the quasiparticle excitations around the nodes. Since the gap nodes lie close to the diagonals of the Brillouin zone, these quasiparticle excitations carry both \hat{a} -axis and \hat{b} -axis currents, then the microwave conductivity should be $CuO₂$ -plane isotropic. However, the ortho-II $YBa₂Cu₃O_{6.50}$ is a very special cuprate SC material and is characterized by a periodic alternative of filled and empty \hat{b} -axis CuO chains.^{8,[9](#page-5-5)} In particular, the recent experimental measurements⁹ unambiguously establish that the \hat{b} -axis CuO chain-induced impurity is the dominant source of the anisotropy in the microwave-conductivity spectrum of the ortho-II $YBa₂Cu₃O_{6.50}$. This experimental result⁹ also implies that the \hat{b} -axis CuO chain in the ortho-II YBa₂Cu₃O_{6.50}-induced impurities lead to an anisotropy of the extended impurityscattering potential in the $CuO₂$ planes. In this paper we show explicitly if the effect of the anisotropy of the extended impurity-scattering potential induced by the \hat{b} -axis CuO chain is considered within the kinetic-energy driven SC mechanism, one can reproduce some main features of the anisotropy in the microwave-conductivity spectrum observed^{8[,9](#page-5-5)} experimentally on the ortho-II YBa₂Cu₃O_{6.50}.

The rest of this paper is organized as follows. In Sec. [II,](#page-1-0) we present the basic formalism, where the BSC-type Green's function under the kinetic-energy driven SC mechanism is dressed via the extended anisotropy impurity scattering. Within this framework, we calculate explicitly the \hat{a} -axis and \hat{b} -axis microwave conductivities based on the nodal approximation of the quasiparticle excitations and scattering processes. The energy and temperature dependences of the \hat{a} -axis and \hat{b} -axis microwave conductivities of the ortho-II $YBa₂Cu₃O_{6.50}$ are presented in Sec. [III.](#page-3-0) Section [IV](#page-4-0) is devoted to a summary.

II. THEORETICAL FRAMEWORK

The basic element of cuprate superconductors is twodimensional $CuO₂$ planes² as mentioned above. It has been shown that the essential physics of the doped $CuO₂$ plane is properly accounted by the t -*J* model on a square lattice, $2,3$ $2,3$

$$
H = -t\sum_{i\hat{\eta}\sigma} C_{i\sigma}^{\dagger} C_{i+\hat{\eta}\sigma} + \mu \sum_{i\sigma} C_{i\sigma}^{\dagger} C_{i\sigma} + J \sum_{i\hat{\eta}} \mathbf{S}_{i} \cdot \mathbf{S}_{i+\hat{\eta}}, \quad (1)
$$

where $\hat{\eta} = \pm \hat{x}, \pm \hat{y}, C^{\dagger}_{i\sigma}$ ($C_{i\sigma}$) is the electron creation (annihilation) operator, $S_i = (S_i^x, S_i^y, S_i^z)$ is spin operator, and μ is the chemical potential. This *t*-*J* model is subject to an important local constraint $\Sigma_{\sigma} C_{i\sigma}^{\dagger} C_{i\sigma} \le 1$ to avoid the double occupancy[,3](#page-5-3) which can be treated properly in analytical calculations within the charge-spin separation (CSS) fermionspin theory, $14,16$ $14,16$ where the constrained electron operators are decoupled as $C_{i\uparrow} = h_{i\uparrow}^{\dagger} S_{i\downarrow}^{\dagger}$ and $C_{i\downarrow} = h_{i\downarrow}^{\dagger} S_{i}^{\dagger}$, with the spinful fermion operator $h_{i\sigma} = e^{-i\Phi_{i\sigma}}h_i$ describes the charge degree of freedom together with some effects of spin-configuration rearrangements due to the presence of the doped charge carrier itself while the spin operator *Si* describes the spin degree of freedom, then the electron local constraint for the single occupancy is satisfied in analytical calculations. In this CSS fermion-spin representation, the t -*J* model [Eq. (1) (1) (1)] can be expressed as

$$
H = t \sum_{i\hat{\eta}} (h_{i+\hat{\eta}|}^{\dagger} h_{i\uparrow} S_i^{\dagger} S_{i+\hat{\eta}}^{\dagger} + h_{i+\hat{\eta}|}^{\dagger} h_{i\downarrow} S_i^{\dagger} S_{i+\hat{\eta}}^{\dagger}) - \mu \sum_{i\sigma} h_{i\sigma}^{\dagger} h_{i\sigma}
$$

+ $J_{\text{eff}} \sum_{i\hat{\eta}} \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{\eta}},$ (2)

with $J_{\text{eff}} = (1 - \delta)^2 J$, and $\delta = \langle h_{i\sigma}^\dagger h_{i\sigma} \rangle = \langle h_i^\dagger h_i \rangle$ is the chargecarrier doping concentration. For a understanding of the SCstate properties of cuprate superconductors, the kineticenergy driven SC mechanism has been developed, $13,14$ $13,14$ where the interaction between charge carriers and spins from the kinetic-energy term in the t -*J* model [Eq. (2) (2) (2)] induces the charge-carrier pairing state with the *d*-wave symmetry by exchanging spin excitations, then the electron Cooper pairs originating from the charge-carrier pairing state are due to the charge-spin recombination, and their condensation reveals the SC ground state. In particular, it has been shown that this SC state is a conventional BCS type with the *d*-wave symmetr[y17](#page-5-15) so that the basic BCS formalism with the *d*-wave SC gap function is still valid in quantitatively reproducing all main low-energy features of the SC coherence of quasiparticles, although the pairing mechanism is driven by the kinetic energy by exchanging spin excitations. Following our previ-

FIG. 1. The anisotropic impurity scattering within the *d*-wave case. V_1 , V_2 , V_3 , and V_4 are potentials for intranode, adjacent-node along the *aˆ*-axis direction, opposite-node, and adjacent-node impurity scattering along the \hat{b} -axis direction scattering.

ous discussions[,12](#page-5-10) the electron Green's function in the SC state can be obtained in the Nambu representation as

$$
\widetilde{G}(\mathbf{k},\omega) = Z_F \frac{\omega \tau_0 + \bar{\Delta}_Z(\mathbf{k}) \tau_1 + \bar{\varepsilon}_{\mathbf{k}} \tau_3}{\omega^2 - E_{\mathbf{k}}^2},\tag{3}
$$

where τ_0 is the unit matrix, τ_1 and τ_3 are Pauli matrices, other notations are defined as same as in Ref. [12](#page-5-10) and have been determined by the self-consistent calculation.^{13[,14](#page-5-12)}

In the presence of impurities, the unperturbed electron Green's function $[Eq. (3)]$ $[Eq. (3)]$ $[Eq. (3)]$ is dressed via the impurity scattering, $18-20$

$$
\widetilde{G}_I(\mathbf{k},\omega) = \sum_{\alpha} \widetilde{G}_{I\alpha} \tau_{\alpha}(\mathbf{k},\omega) = [\widetilde{G}(\mathbf{k},\omega)^{-1} - \widetilde{\Sigma}(\mathbf{k},\omega)]^{-1}, (4)
$$

with the self-energy $\tilde{\Sigma}(\mathbf{k}, \omega) = \sum_{\alpha} \sum_{\alpha} (\mathbf{k}, \omega) \tau_{\alpha}$. It has been shown that all but the scalar component of the self-energy function can be neglected or absorbed into $\overline{\Delta}_z(\mathbf{k})$.^{[18](#page-5-16)[–20](#page-5-17)} In this case, the dressed electron Green's function $[Eq. (4)]$ $[Eq. (4)]$ $[Eq. (4)]$ can be explicitly rewritten as

$$
\widetilde{G}_I(\mathbf{k},\omega) = Z_F \frac{[\omega - \Sigma_0(\mathbf{k},\omega)]\tau_0 + \bar{\Delta}_Z(\mathbf{k})\tau_1 + [\bar{\varepsilon}_{\mathbf{k}} + \Sigma_3(\mathbf{k},\omega)]\tau_3}{[\omega - \Sigma_0(\mathbf{k},\omega)]^2 - \bar{\varepsilon}_{\mathbf{k}}^2 - \bar{\Delta}_Z^2(\mathbf{k})}
$$
\n(5)

where the self-energies $\Sigma_0(\mathbf{k}, \omega)$ and $\Sigma_3(\mathbf{k}, \omega)$ are treated within the framework of the *T*-matrix approximation as

$$
\widetilde{\Sigma}(\mathbf{k},\omega) = \rho_i \widetilde{T}_{\mathbf{k}\mathbf{k}}(\omega),\tag{6}
$$

where ρ_i is the impurity concentration and $\tilde{T}_{kk}(\omega)$ is the diagonal element of the *T* matrix,

$$
\widetilde{T}_{\mathbf{k}\mathbf{k}'}(\omega) = V_{\mathbf{k}\mathbf{k}'}\tau_3 + \sum_{\mathbf{k}''} V_{\mathbf{k}\mathbf{k}''}\tau_3 \widetilde{G}_I(\mathbf{k}'', \omega) \widetilde{T}_{\mathbf{k}''\mathbf{k}'}(\omega), \qquad (7)
$$

with $V_{kk'}$ is the impurity-scattering potential and $\tilde{T}_{kk'}(\omega)$
= $T_{kk'}^0(\omega)\tau_0 + T_{kk'}^3(\omega)\tau_3$. In our earlier work without considering the \hat{b} -axis CuO chain-induced impurities,¹² we have discussed the effect of the extended isotropy impurityscattering potential on the quasiparticle transport in the nonortho-II phase of cuprate superconductors within the nodal approximation of quasiparticle excitations and scattering processes, where there is no gap to the quasiparticle excitations at the four nodes, and then the quasiparticles are generated only around these four nodes. In this $CuO₂$ -plane isotropic case, a general scattering potential $V_{kk'}$ need only to be evaluated in three possible cases: $18,19$ $18,19$ the intranode impurity scattering $V_{\mathbf{k}\mathbf{k}'}=V_1$ (**k** and **k**' at the same node), the adjacent-node impurity scattering $V_{kk'}=V_2$ (**k** and **k**^{*'*} at the adjacent nodes), and the opposite-node impurity scattering $V_{\mathbf{k}\mathbf{k}'} = V_3$ (**k** and **k**' at the opposite nodes). However, this $CuO₂$ -plane isotropic case is broken in the presence of the \hat{b} -axis CuO chain-induced impurities in the ortho-II $YBa₂Cu₃O_{6.50}$ since the CuO chain-induced impurityscattering potential can be described by an anisotropic potential in the $CuO₂$ planes.²¹ This simply assumes that the screened Coulomb potential created by the CuO chaininduced impurities produces a footprint sensed by quasiparticles moving in the $CuO₂$ planes.²¹ After incorporating this *b ˆ*-axis CuO chain-induced anisotropy impurity scattering into the extended isotropy impurity-scattering potential, the total scattering potential $V_{kk'}$ need to be evaluated in four possible cases as shown in Fig. [1:](#page-1-5) the intranode impurity scattering $V_{kk'}=V_1$ (**k** and **k**' at the same node) and the opposite-node impurity scattering $V_{kk'}=V_3$ (**k** and **k**^{*'*} at the

opposite nodes), these two cases are the same as in the previous discussions in the nonortho-II phase of cuprate superconductors.¹² However, the adjacent-node impurity scattering along the \hat{a} -axis direction $V_{kk'}=V_2$ (**k** and **k**' at the \hat{a} -axis adjacent nodes) is different from that along the \hat{b} -axis direction $V_{kk'}=V_4$ (**k**, and **k**' at the \hat{b} -axis adjacent nodes) in the present discussions of the ortho-II $YBa₂Cu₃O_{6.50}$. In this case, the impurity-scattering potential $V_{kk'}$ in the *T* matrix is effectively reduced as

$$
V_{kk'} \rightarrow V = \begin{pmatrix} V_1 & V_2 & V_3 & V_4 \\ V_2 & V_1 & V_4 & V_3 \\ V_3 & V_4 & V_1 & V_2 \\ V_4 & V_3 & V_2 & V_1 \end{pmatrix} .
$$
 (8)

We emphasize that the anisotropy of the impurityscattering potential (then $CuO₂-plane$ microwave conductivity) has been reflected by this important difference between the adjacent-node impurity-scattering strengths V_2 and V_4 . Now we follow our previous discussions 12 and obtain explicitly the \hat{a} -axis and \hat{b} -axis microwave conductivities of the ortho-II YBa₂Cu₃O_{6+*v*} as

$$
\sigma_a(\omega, T) = 4e^2v_f^2 \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{n_F(\omega') - n_F(\omega' + \omega)}{\omega'} \left[\text{Re } J_a(\omega' - i\delta, \omega' + \omega + i\delta) - \text{Re } J_a(\omega' + i\delta, \omega' + \omega + i\delta) \right],\tag{9a}
$$

$$
\sigma_b(\omega, T) = 4e^2v_f^2 \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{n_F(\omega') - n_F(\omega' + \omega)}{\omega'} \left[\text{Re } J_b(\omega' - i\delta, \omega' + \omega + i\delta) - \text{Re } J_b(\omega' + i\delta, \omega' + \omega + i\delta) \right],\tag{9b}
$$

where $n_F(\omega)$ is the fermion distribution function, $v_f = \sqrt{2}t$ is the electron velocity at the nodal points, and the kernel function $J_a(\omega', \omega)$ and $J_b(\omega', \omega)$ are given by

$$
J_a(\omega', \omega) = \frac{I_0^{(0)} + L_1^a [I_0^{(0)} I_3^{(3)} + I_0^{(3)} I_3^{(0)}]}{[1 - (L_1^a I_0^{(0)} + L_2^a I_3^{(0)})][1 - (L_1^a I_3^{(3)} + L_2^a I_0^{(3)})] - [L_1^a I_0^{(3)} + L_2^a I_3^{(3)}][L_1^a I_3^{(0)} + L_2^a I_0^{(0)}]},
$$
\n(10a)

$$
J_b(\omega', \omega) = \frac{I_0^{(0)} + L_1^b [I_0^{(0)} I_3^{(3)} + I_0^{(3)} I_3^{(0)}]}{[1 - (L_1^b I_0^{(0)} + L_2^b I_3^{(0)})][1 - (L_1^b I_3^{(3)} + L_2^b I_0^{(3)})] - [L_1^b I_0^{(3)} + L_2^b I_3^{(3)}][L_1^b I_3^{(0)} + L_2^b I_0^{(0)}]},
$$
\n(10b)

where the functions L_1^a , L_2^a , L_1^b , and L_2^a are expressed as

$$
L_1^a(\omega', \omega) = \rho_i [T_{11}^0(\omega') T_{11}^0(\omega' + \omega) + T_{11}^3(\omega') T_{11}^3(\omega' + \omega) - T_{12}^0(\omega') T_{12}^0(\omega' + \omega) - T_{12}^3(\omega') T_{12}^3(\omega' + \omega) - T_{13}^0(\omega') T_{13}^0(\omega' + \omega) - T_{13}^3(\omega') T_{13}^3(\omega' + \omega) + T_{14}^0(\omega') T_{14}^0(\omega' + \omega) + T_{14}^3(\omega') T_{14}^3(\omega' + \omega)],
$$
\n(11a)

$$
L_2^a(\omega', \omega) = \rho_i [T_{11}^0(\omega') T_{11}^3(\omega' + \omega) + T_{11}^3(\omega') T_{11}^0(\omega' + \omega) - T_{12}^0(\omega') T_{12}^3(\omega' + \omega) - T_{12}^3(\omega') T_{12}^0(\omega' + \omega) - T_{13}^0(\omega') T_{13}^3(\omega' + \omega) - T_{13}^3(\omega') T_{13}^0(\omega' + \omega) + T_{14}^0(\omega') T_{14}^3(\omega' + \omega) + T_{14}^3(\omega') T_{14}^0(\omega' + \omega)],
$$
\n(11b)

$$
L_1^b(\omega', \omega) = \rho_i [T_{11}^0(\omega') T_{11}^0(\omega' + \omega) + T_{11}^3(\omega') T_{11}^3(\omega' + i\omega) + T_{12}^0(\omega') T_{12}^0(\omega' + \omega) + T_{12}^3(\omega') T_{12}^3(\omega' + \omega) - T_{13}^0(\omega') T_{13}^0(\omega' + \omega) - T_{14}^3(\omega') T_{14}^3(\omega' + \omega) - T_{14}^3(\omega') T_{14}^0(\omega' + \omega) - T_{14}^3(\omega') T_{14}^3(\omega' + \omega) \Big],
$$
\n(11c)

$$
L_2^b(\omega', \omega) = \rho_i [T_{11}^0(\omega') T_{11}^3(\omega' + \omega) + T_{11}^3(\omega') T_{11}^0(\omega' + \omega) + T_{12}^0(\omega') T_{12}^3(\omega' + \omega) + T_{12}^3(\omega') T_{12}^0(\omega' + \omega) - T_{13}^0(\omega') T_{13}^3(\omega' + \omega) - T_{13}^3(\omega') T_{13}^0(\omega' + \omega) - T_{14}^0(\omega') T_{14}^3(\omega' + \omega) - T_{14}^3(\omega') T_{14}^0(\omega' + \omega)],
$$
\n(11d)

and the functions $I_0^{(0)}(\omega, \omega')$ and $I_3^{(0)}(\omega, \omega')$ are evaluated in terms of the dressed Green's function $[Eq. (5)]$ $[Eq. (5)]$ $[Eq. (5)]$ as

$$
I_0^{(0)}(\omega,\omega')\tau_0 + I_0^{(3)}(\omega,\omega')\tau_3 = \frac{1}{N} \sum_{\mathbf{k}} \widetilde{G}_I(\mathbf{k},\omega) \widetilde{G}_I(\mathbf{k},\omega + \omega'),
$$
\n(12a)

$$
I_3^{(0)}(\omega,\omega')\tau_0 + I_3^{(3)}(\omega,\omega')\tau_3 = \frac{1}{N} \sum_{\mathbf{k}} \widetilde{G}_I(\mathbf{k},\omega)\widetilde{\tau}_3 \widetilde{G}_I(\mathbf{k},\omega+\omega').
$$
\n(12b)

It is clearly that if the effect of the \hat{b} -axis CuO chain-induced impurities is neglected, i.e., $V_2 = V_4$, this leads to $L_1^a(\omega', \omega)$ $=L_1^b(\omega', \omega)$ and $L_2^a(\omega', \omega) = L_2^b(\omega', \omega)$, and then the *â*-axis and \hat{b} -axis microwave conductivities in Eq. ([9](#page-2-0)) are reduced to the isotropic one¹² $\sigma_a(\omega', \omega) = \sigma_b(\omega', \omega) = \sigma(\omega', \omega)$.

III. ENERGY AND TEMPERATURE DEPENDENCES OF THE *aˆ***-AXIS AND** *b ˆ***-AXIS MICROWAVE CONDUCTIVITIES FOR THE ORTHO-II YBa₂Cu₃O_{6.50}**

In cuprate superconductors, although the values of *J* and *t* are believed to vary somewhat from compound to compound, $²$ however, as a qualitative discussion, the com-</sup> monly used parameters in this paper are chosen as $t/J = 2.5$, with an reasonably estimative value of $J \sim 1000$ K. We are now ready to discuss the energy and temperature dependences of the \hat{a} -axis and \hat{b} -axis quasiparticle transport of the ortho-II $YBa₂Cu₃O_{6.50}$ with the extended anisotropy impurity scattering. We have performed a calculation for the energy dependence of the \hat{a} -axis and \hat{b} -axis microwave conductivities $\sigma_a(\omega, T)$ and $\sigma_b(\omega, T)$ in Eq. ([9](#page-2-0)) at low temperatures, and the results of $\sigma_a(\omega, T)$ (top panel) and $\sigma_b(\omega, T)$ (bottom panel) as a function of energy with temperature $T=0.002J$ $= 2$ K (solid line), $T = 0.004J = 4$ K (dash-dotted line), and $T=0.008J=8$ K (dashed line) under the slightly strong impurity-scattering potential with $V_1 = 100J$, $V_2 = 90J$, V_3 $= 30J$, and $V_4 = 30J$ at the impurity concentration ρ_i $= 0.00002$ for the doping concentration $\delta = 0.15$ are plotted in Fig. [2](#page-3-1) in comparison with the corresponding experimental results⁸ of the ortho-II YBa₂Cu₃O_{6.50} (inset). It is clearly that the anisotropy of the energy evolution of the lowtemperature microwave conductivity of the ortho-II $YBa₂Cu₃O_{6.50}$ is qualitatively reproduced,^{8,[9](#page-5-5)} where quasiparticle spectral weights differ by a factor of 2 between the \hat{a} -axis and \hat{b} -axis directions. Moreover, although the cusplike line shapes are observed along both \hat{a} -axis and \hat{b} -axis directions as in the previous case in the nonortho-II phase of cuprate superconductors[,12](#page-5-10) the width of the *aˆ*-axis spectrum is significantly broadened. This broadening is only attributed to increased quasiparticle scattering arising from the CuO chain-induced impurities. In comparison with our

FIG. 2. The microwave conductivities of the \hat{a} -axis (top panel) and \hat{b} -axis (bottom panel) as a function of energy with $T=0.002J$ $= 2$ K (solid line), $T=0.004J=4$ K (dash-dotted line), and *T*= 0.008*J* = 8 K (dashed line) at ρ_i = 0.00002 for *t*/*J* = 2.5 with *V*₁ = 100*J*, *V*₂ = 90*J*, *V*₃ = 30*J*, and *V*₄ = 30*J* in δ = 0.15. Inset: the corresponding experimental result of the ortho-II $YBa₂Cu₃O_{6.50}$ taken from Ref. [8.](#page-5-7)

previous results for the nonortho-II phase of cuprate superconductors, 12 the present results of the anisotropy therefore confirm that the CuO chain-induced impurity is the dominant source of the anisotropy in the microwaveconductivity spectrum of the ortho-II $YBa₂Cu₃O_{6.50}$.

In the above discussions of the low-temperature and lowenergy cases, the inelastic quasiparticle-quasiparticle scattering process has been dropped since it is suppressed at low temperatures and low energies due to the large SC gap parameter in the quasiparticle excitation spectrum. For a better understanding of the anisotropy in the microwaveconductivity spectrum, we now turn to discuss the temperature dependence of the quasiparticle transport of the ortho-II $YBa₂Cu₃O_{6.50}$, where *T* may approach to T_c from lowtemperature side and therefore the inelastic quasiparticlequasiparticle scattering process should be considered[.19](#page-5-18) The contribution from this inelastic quasiparticle-quasiparticle scattering process is increased rapidly when *T* approaches to T_c from low-temperature side since there is a small SC gap parameter near T_c . In particular, it has been pointed out²² that the contribution from the quasiparticle-quasiparticle scattering process to the transport lifetime is exponentially suppressed at low temperatures, and therefore the effect of this inelastic quasiparticle-quasiparticle scattering can be considered by adding the inverse transport lifetime²⁰ $\tau_{\text{inel}}^{-1}(T)$ to the imaginary part of the self-energy function $\Sigma_0(\omega)$ in Eq. ([5](#page-1-6)) as in our previous discussions, 12 then the total self-energy function $\Sigma_0^{\text{tot}}(\omega)$ can be expressed as^{12,[19](#page-5-18)[,22](#page-5-20)} $\Sigma_0^{\text{tot}}(\omega) = \Sigma_0(\omega)$ $-i[2\tau_{\text{inel}}(T)]^{-1}$, with $\tau_{\text{inel}}(T)$ has been chosen as $[2\tau_{\text{inel}}(T)]^{-1} = 2 \times 10^4 (T - 0.002)^4 J$. Using this total selfenergy function $\Sigma_0^{\text{tot}}(\omega)$ to replace $\Sigma_0(\omega)$ in Eq. ([5](#page-1-6)), we have performed a calculation for the temperature dependence of the \hat{a} -axis and \hat{b} -axis microwave conductivities $\sigma_a(\omega, T)$ and $\sigma_b(\omega, T)$, and the results of $\sigma_a(\omega, T)$ and $\sigma_b(\omega, T)$ as a function of temperature *T* with energy $\omega = 0.0000547J$ \approx 1.14 GHz (solid line), ω =0.0001094*J* \approx 2.28 GHz (dashdotted line), ω =0.0006564*J* \approx 13.4 GHz (dashed line), and ω =0.001094*J* \approx 22.8 GHz (dotted line) under the slightly strong impurity-scattering potential with $V_1 = 100J$, $V_2 = 90J$, $V_3 = 30J$, and $V_4 = 30J$ at the impurity concentration ρ_i = 0.00002 for the doping concentration δ = 0.15 are plotted in Fig. [3.](#page-4-1) For comparison, the corresponding experimental results⁸ of the ortho-II YBa₂Cu₃O_{6.50} are also plotted in Fig. [3](#page-4-1) (inset). Obviously, the overall temperature dependence is similar to that of the temperature dependence of the microwave conductivity in the nonortho-II phase of cuprate superconductors,¹² where both \hat{a} -axis and \hat{b} -axis temperature-dependent microwave conductivities $\sigma_a(\omega, T)$ and $\sigma_b(\omega, T)$ increases rapidly with increasing temperatures to a broad peak, and then falls roughly linearly. In particular, this broad peak shifts to higher temperatures as the energy increased. However, the spectral width is evidently anisotropic, where the spectral width of the *aˆ*-axis microwave conductivity $\sigma_a(\omega, T)$ is larger than the corresponding value of the \hat{b} -axis one. Moreover, the peak position of the \hat{a} -axis microwave conductivity $\sigma_a(\omega, T)$ is located at higher temperature than the corresponding value of the \hat{b} -axis one, in qualitative agreement with the experimental data of the ortho-II YBa₂Cu₃O_{6.50}.^{[8](#page-5-7)[,9](#page-5-5)}

In our present theory, the anisotropy of the microwave conductivity reflects directly from the electron vertex correction due to the extended anisotropy impurity-scattering potential. However, it has been shown that for all scattering strength the thermal vertex correction is negligible compared to the electron one, 18 therefore we expects the heat transport of the ortho-II YBa₂Cu₃O_{6.50} to be CuO₂-plane isotropic. This is a unique feature of our theory that is different from the other theory¹⁰ in which the Fermi surface is assumed to be altered and thus the heat transport may be also anisotropic. This should be verified by further experiments.

FIG. 3. The microwave conductivities of the \hat{a} axis (top panel) and \hat{b} axis (bottom panel) as a function of temperature with energy $\omega \approx 1.14$ GHz (solid line), $\omega \approx 2.28$ GHz (dash-dotted line), $\omega \approx 13.4$ GHz (dashed line), and $\omega \approx 22.8$ GHz (dotted line) at $\rho_i = 0.00002$ for $t/J = 2.5$ with $V_1 = 100J$, $V_2 = 90J$, $V_3 = 30J$, and V_4 =30*J* in δ =0.15. Inset: the corresponding experimental result of the ortho-II YBa₂Cu₃O_{6.50} taken from Ref. [8.](#page-5-7)

IV. SUMMARY

Within the framework of the kinetic-energy driven SC mechanism, we have studied the anisotropy in the microwave-conductivity spectrum recently observed^{8[,9](#page-5-5)} in the ortho-II YBa₂Cu₃O_{6.50}. The ortho-II YBa₂Cu₃O_{6.50} is characterized by a periodic alternative of filled and empty \hat{b} -axis CuO chains. The extended anisotropy impurity-scatteringpotential results when the extended isotropy impurityscattering potential in the $CuO₂$ planes incorporates the \hat{b} -axis CuO chain-induced impurity-scattering potential.

Based on the nodal approximation of the quasiparticle excitations and scattering processes, we have calculated the \hat{a} -axis and \hat{b} -axis microwave conductivities with the extended anisotropy impurity scattering, and reproduced the main features 8.9 of the anisotropy in the microwaveconductivity spectrum of the ortho-II YBa₂Cu₃O_{6.50}, including the intensity and line shape of the energy and temperature dependences of the \hat{a} -axis and \hat{b} -axis microwave

conductivities. Our results also confirm that the \hat{b} -axis CuO chain-induced impurity is the main source of the anisotropy.⁹

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*Author to whom correspondence should be addressed.

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